

## Negative differential magnetoresistance and commensurability oscillations of two-dimensional electrons in a disordered array of antidots

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**Abstract.** Negative linear magnetoresistance of two-dimensional (2D) electrons has been found in a disordered array of antidots. We suggest that trajectories that roll along the array of antidots exist in a magnetic field. These trajectories have a mean free path larger than the average value for electrons with ordinary diffusion.

A magnetic field decreases the mean free path of electrons, therefore the magnetoresistance of semiconductors and metals is positive. However, at low temperature and weak magnetic field negative magnetoresistance has been observed [1]. This phenomenon has been explained from a quantum standpoint. The crossed electron trajectories increase the backscattering probability due to the interference at the crossing point. Magnetic field suppresses this interference because of the Aharonov–Bohm effect, the backscattering probability decreases, and as a result negative magnetoresistance appears [1]. From the classical standpoint the negative magnetoresistance is a result of the increase of the electron elastic scattering length or the appearance of carriers with a mean free path larger than the average value. In [2] Wagenhuber *et al* have found theoretically that this probability exists in a two-dimensional (2D) electron gas in a lateral 2D lattice. The anomalous diffusion of electrons in a magnetic field was predicted. This is characterized by a linear increase of the mean square displacement of carriers  $\langle x^2 \rangle^{1/2} \simeq t^{1+\alpha}$  with  $\alpha = 1$ . It is responsible for the violation of the exponential distribution of a portion of particles with mean free path, and thus, electrons with an anomalously large elastic scattering length appear. However, the influence of this effect on the electron transport, and in particular on the magnetoresistance, was not considered.

One type of 2D lateral superlattice with a strong repulsive potential is an array of antidots [4], holes with submicron diameter produced in a 2D electron gas by etching or irradiation by ions. This system has also attracted attention because it allows various arrangements of antidots to be simulated: square [3, 4], hexagonal [5], two-dimensional quasicrystals [6], and disordered [7, 8].

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Table 1. The mean free path as a function of the degree of disorder.

$\Delta$ ( $\mu\text{m}$ )	$l$ before illumination ( $\mu\text{m}$ )	$l$ after illumination ( $\mu\text{m}$ )
0.0	0.4	0.41
0.1	0.17	0.18
0.25	0.21	0.39
0.35	0.39	0.45

In this work the magnetoresistance of 2D electrons in periodic and disordered lattices of antidots has been studied, and the negative magnetoresistance, which grows with degree of disorder, has been found.

The test samples were Hall bridges based on GaAs/AlGaAs heterostructures with a 2D electron gas. The parameters of the initial heterostructures were: electron density  $n_s = 5 \times 10^{11} \text{ cm}^{-2}$ ; mobility  $\mu = (2 - 5) \times 10^5 \text{ cm}^2 \text{ V s}$ . A lattice of antidots, produced by electron-beam lithography and reactive ion etching, covered a part of the sample between the potential probes. The antidot diameter was  $0.15\text{--}0.2 \mu\text{m}$ . The effective antidot diameter  $a$ , which consists of a lithographic diameter and a depletion length, was  $0.2\text{--}0.3 \mu\text{m}$ . Samples with various degrees of disorder of the array of antidots were made for experiment (figure 1). The disordering of the lattice was accomplished in the following way. The random-number generator determined the shift in the position of the antidots in the direction of the neighbouring antidots. The deviations of antidots from their periodic arrangement in the lattice with period  $d = 0.7 \mu\text{m}$  at the peak were  $\Delta = 0.0, 0.1, 0.25$  and  $0.35 \mu\text{m}$ . Thus, the short-range order of the system was violated, but its long-range order was preserved.

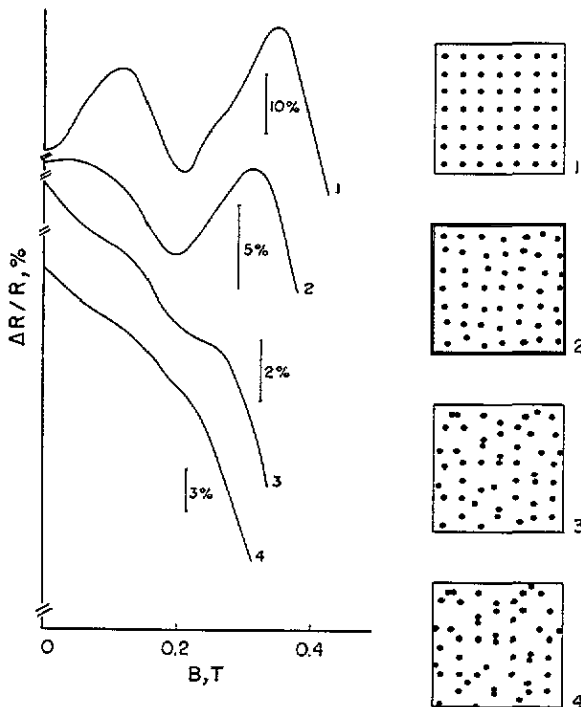


Figure 1. Magnetoresistance dependence on  $B$  for a sample with different degrees of antidot disorder and for a periodic lattice:  $d = 0.7 \mu\text{m}$ ; 1,  $\Delta = 0.0$ ; 2,  $\Delta = 0.1$ ; 3,  $\Delta = 0.25$ ; 4,  $\Delta = 0.35 \mu\text{m}$ ;  $T = 4.2 \text{ K}$ .

The magnetoresistance was measured using the four-terminal method at frequencies 70–700 Hz in a magnetic field of up to 0.4 T and at temperatures of 1.7–4.2 K. Electron scattering by the antidot lattice was dominant in our samples; from this the mean free path  $l$  (which was determined from the resistance at  $B = 0$ ) was found to be 10 times lower than in samples without antidots. Table 1 shows the value of  $l$  for samples with various degrees of disorder. Surprisingly, the behaviour of  $l$  is non-monotonic with disorder. We see, that with an increase of  $\Delta$ , the length  $l$  initially decreases, and then increases again. Similar non-monotonic behaviour has been observed recently in samples with a quasiperiodic (Penrose tiling) lattice of antidots [6], which is an intermediate type of array between periodic and disordered. In this case the length  $l$  initially decreases with length increase of the basic triangle in the Penrose cell, and then increases again. We believe that this non-monotonic behaviour of  $l$  with disorder is not connected with inhomogeneity or variation of mobility in the initial samples before patterning of antidots. We fabricated several samples with the same periodicity and found no difference in the apparent mobility. We also found that the apparent mobility in a system with a periodic antidot lattice is proportional to the distance  $d-a$  between antidots. Thus, the non-monotonic behaviour of  $l$  in a disordered antidot lattice has not been explained, and further theoretical analysis of the scattering in this system is required.

Figure 1 shows the magnetoresistance for samples with various degrees of disorder as a function of magnetic field. We see for all samples commensurability oscillations when the cyclotron diameter  $R_L$  is comparable to the lattice period. The oscillation amplitude decreases with increase of  $\Delta$ , and the position of last peak shifts to lower magnetic field (this shift is more clearly seen in larger-scale figures [8]). The long-range order preservation in our sample is responsible for oscillations in the disordered antidot lattice. Figure 1 shows that for the periodic lattice, the second peak maximum is higher than the peak maximum that lies at lower magnetic field. However, for the array with degree  $\Delta = 0.1 \mu\text{m}$  of disorder their amplitudes are equal, i.e. the negative magnetoresistance (nMR) appears, but it is not visible due to oscillation. With increasing degree of disorder nMR at low  $B$  is observed. Figure 2 shows the nMR low-field part for samples with a disordered array of antidots. We see that the magnetoresistance is linear for low magnetic field and increases with growth of disorder. We did not find this magnetoresistance to be temperature dependent at temperatures of 1.7–4.2 K, which proves that this nMR has a classical origin. We should note that for weak magnetic field up to 10 mT on nMR ( $\Delta R/R \simeq 0.2\%$ ) dependence on  $R$  has been found [7], which can be described by the expression for weak localization corrections to the conductivity. Approximation of this magnetoresistance to the higher magnetic field  $B = 0.1$  T gives the value  $\Delta R/R \simeq 1\%$  compared to the value of observed magnetoresistance. However, it should give rise to nMR deviation from linear  $B$  dependence and contribute with a  $T$  dependent part, which was not found in experiments. We should note that electrons in an antidot array move ballistically from one antidot to another and, therefore, a low magnetic field ( $\sim 0.01$  T) influences their motion; in particular, this is responsible for the commensurability oscillations [4]. In this case the weak localization corrections to the conductivity theory cannot be used because it includes only interference by magnetic field diffusive trajectories with unperturbed trajectories. In addition, the monotonic increase of linear nMR with degree of disorder (figure 2) also gives evidence that this nMR is not connected with weak localization corrections to the conductivity, because it has a non-monotonic behaviour as a function of  $\Delta$ , in correlation with resistivity [7].

For Hall-bridge samples  $\rho_{xx} = \sigma_{xx}/(\sigma_{xx}^2 + \sigma_{xy}^2)$ , where  $\rho_{xx}$  is the resistivity, and  $\sigma_{xx}$  and  $\sigma_{xy}$  are the diagonal and Hall parts, respectively, of the conductivity.  $\rho_{xx}$  should be a constant in a magnetic field for one group of the charge carriers. Thus, in our case the

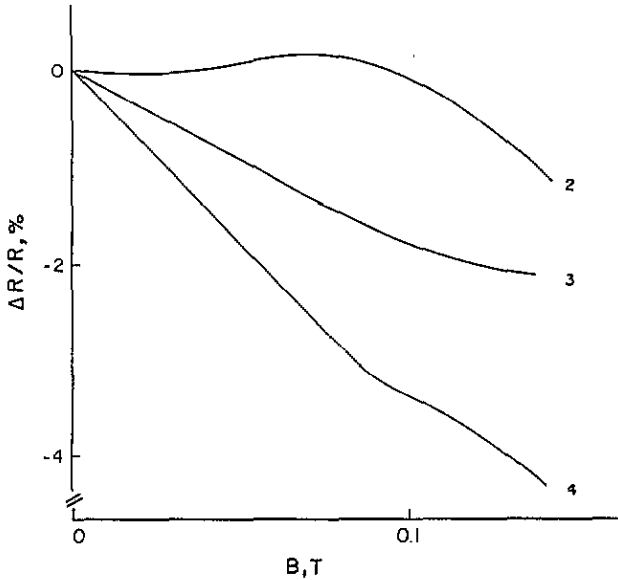


Figure 2. Magnetoconductance dependence on  $B$  for low magnetic field: 2,  $\Delta = 0.1$ ; 3,  $\Delta = 0.25$ ; 4,  $\Delta = 0.35 \mu\text{m}$ ;  $T = 4.2 \text{ K}$ .

appearance of nMR at low magnetic field is a result of a total conductivity increase; however, the coexisting Drude contribution decreases the total conductivity:

$$\sigma_{xx} = \sigma_0 / [1 + (\omega_c \tau)^2]$$

where  $\omega_c$  is the cyclotron frequency,  $\tau$  is the elastic scattering time, and  $\sigma_0$  is the Drude conductivity at zero magnetic field. In our case for a weak magnetic field the Drude contribution to the conductivity is small, and the observed nMR corresponds to a positive magnetoconductance  $\Delta\sigma_{xx} > 0$ , although at stronger magnetic field  $\Delta\sigma_{xx} < 0$ . The Drude conductivity and  $\sigma_{xy}$  are exactly compensated, and we observe only nMR.

In a one-dimensional lateral superlattice with strong modulated potential a positive magnetoconductance has been observed [9, 10]. This was analysed from the classical standpoint and explained by the formation of open electron orbits, which drift in the crossed external magnetic field and in the periodic superlattice electric field. For the periodic antidot lattice, as mentioned above, at low magnetic field commensurability oscillations have been found. With increase of  $B$ , when  $2R_L < d$  nMR has been observed [3, 4]. Increase of electrons scattered by antidots and localization around antidots are responsible for this magnetoconductance. In this case the magnetic field is strong, and  $\rho_{xx} \simeq \rho_{xy}$ , where  $\rho_{xy}$  is the Hall resistivity. Thus, because  $\rho_{xx} \simeq \rho_{xy}$ , a decrease in the resistivity signifies a decrease in the conductivity, i.e. localization, in contrast to the weak-magnetic-field case, where  $\rho_{xx} \simeq 1/\sigma_{xx}$ .

There are two models that explain commensurability oscillations in an antidot lattice. The first model considers electron orbits that are not scattered by antidots [4]. A portion  $f_p$  of these orbits oscillates as a function of magnetic field, but the amplitude of these oscillations is too small to explain the conductivity oscillations. The second model was suggested by Fleischmann *et al* [11]. They calculated the contribution of the chaotic trajectories that lie in the phase volume near the regular pinned orbits. The resultant conductivity is [11]

$$\sigma \simeq (1 - f_p) \int dt \exp(-t/\tau) (v_i(t) v_j(0))$$

where  $\langle v_i(x)v_j(0) \rangle$  is the velocity correlation function. Fleischmann *et al* have found that the main contribution to the conductivity is determined by the chaotic orbits, which are localized around antidot groups. Baskin *et al* [12] also considered the electron motion in the antidot lattice from the standpoint of the dynamic chaos theory, but in this model stable trajectories that roll along the lattice row have been found. These trajectories, in contrast to the localized orbits considered in [11], increase the total conductivity. However, the contribution of these trajectories to the conductivity is not sufficient to explain the oscillation amplitudes. Therefore nearby chaotic trajectories with the same dynamics should be included in the electron diffusion coefficient calculation. The contribution of the runaway trajectories to the conductance can be found from the theory for resistance of point contacts [12]:

$$\sigma = (2e^2/h)[k_F(d-a)/\pi](L/d)(2S/\pi^2)$$

where  $S$  is the fraction of the phase space filled by runaway trajectories,  $L$  is the length of the sample, or  $L = l$ , and  $k_F$  is the electron wave vector. The diffusion coefficient of the quasi-runaway trajectories was calculated through a numerical simulation. Thus, one model analyses an 'island of pinned trajectories' and a nearby sea of chaotic orbits with the same dynamics [11], and the other an 'island of runaway trajectories' surrounded by the sea of electron orbits with anomalous diffusion [12]. Agreement of theory with experiment without adjustable parameters has been obtained in [11], therefore the contribution of chaotic orbits to conductivity is dominant. The existence of commensurability oscillations in a disordered antidot lattice (figure 1), as indicated above, gives evidence of the stability of pinned electron trajectories when the short-range order is violated. The reason can be connected to the fact that the chaotic orbits are localized within several superlattice periods. In a disordered antidot array a long-range order is preserved, therefore these orbits are not destroyed in contrast to the regular trajectories which are determined by short-range order. It should be noted that the runaway trajectories considered in model [12] have a diffusion coefficient divergency, since the relation  $\langle x(t) \rangle \simeq t$  holds for these electrons. Similar trajectories with anomalous diffusion have been predicted in a superlattice with a weak periodic potential [2]. These orbits appear in a weak magnetic field. For the antidot array the same situation occurs if  $d/a \gg 1$ ; however at finite magnetic field the portion of runaway trajectories should oscillate as a function of  $B$ . For the samples investigated in our work, the ratio  $d/a = 3.5$ , and trajectories with anomalous diffusion at low magnetic field are not prevented by shadowing antidots in the next rows. The situation is different for a disordered antidot lattice. In this case runaway trajectories could appear not only for commensurability conditions, as in the periodic system, but at any magnetic field. Two factors are responsible for the existence of anomalous diffusion trajectories: correlation of the electron cyclotron radius, and distance between antidots in any magnetic field because of disorder and accidental absence of shadowing of these orbits. Thus, the NMR observed in the disordered antidot lattice could be due to the existence of trajectories with anomalous diffusion in the magnetic field. Also the probability of appearance of these orbits augments with increase of disorder, and growth of the magnetoresistance value is expected, as observed in experiments (figure 2). The runaway trajectory is an analogy with the edge states in a classical strong magnetic field [13]. The conductance of these trajectories can be written in the form

$$\sigma = e^2 N/h$$

where  $N \simeq \frac{1}{2}k_F R_L$ , thus

$$\sigma \simeq B^{-1}$$

however, conductivity decreases with  $B$ . Recently the linear magnetoresistance in the quantum point contact has been observed; it was negative due to a difference in the value of the edge current inside and Hall current outside the contact [14]. Our situation is similar; however the difference between the Hall and the diagonal components of anomalous currents is not clear. It should be noted that recently nMR in periodic lattices of antidots and dots has been observed [15]. However, the modulation potential in this superlattice was four times less than the Fermi energy, in contrast to our case, when the antidots have a strong repulsive potential. For a weak modulation potential, trajectories that do not contribute to the current  $B = 0$  were suggested [15]. In our case all electrons bounce like balls ballistically through the antidot lattice and contribute to the diffusion coefficient.

In summary, the samples with a disordered antidot lattice, in contrast to the periodic array, have revealed linear nMR, which is temperature independent. Against the magnetoresistance background commensurability oscillations due to chaotic orbit localization have been found. We believe that the trajectories that roll along rows of the lattice and pinned orbits remain stable when the short-range lattice order is violated. The small ratio  $d/a$  is responsible for this stability. Further antidot lattice experimental work with larger ratio  $d/a$  and theoretical analysis of the transition from order to disorder are required.

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